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Technical Note

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Structure of the Canonical Turbulent Wall-Bounded Flow

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I. Introduction

ONE hundred years after Ludwig Prandtl's fundamental lecture on boundary layer theory [1], the mean-velocity profile and the shear-stress distribution of the seemingly simplest case of wall-bounded flow, the zero-pressure-gradient turbulent boundary layer (ZPG TBL), appears to be still terra incognita. Less is even known about confined and semiconfined flows subjected to pressure gradients, such as pipe and channel flows and wall-bounded flows approaching pressure-driven separation. The problem is of course related to the lack of analytical solutions to the instantaneous, nonlinear Navier–Stokes equations that govern the stochastic dependent variables of almost all turbulent flows. What little we know about turbulence comes from experiments and heuristic modeling, not first-principles solutions. [Direct numerical simulations (DNS) provide first-principles integration of the instantaneous Navier–Stokes equations, but is at present limited to modest Reynolds numbers and simple geometries.] A first step to properly describe wall-bounded turbulent flows is to form a general idea of the structure of this type of flow.

In a laminar boundary layer, a viscous region exists within the entire boundary layer and an inviscid region exists outside. A turbulent boundary layer, in contrast, is infinitely more complex. Within the boundary layer itself more than one region, with varying degrees of importance for viscosity, could be identified. In between these layers, so-called overlap layers may occur. Based on experimental investigations and theoretical considerations, several models of the structure of turbulent wall-bounded layers have been proposed over the years. The classical approach advances a two-layer structure with a single overlap layer (Millikan [2]). Alternative approaches suggest a three-layer structure with two overlap layers (see, e.g., Afzal and Bush [3], Sreenivasan and Sahay [4]). More recently a four-layer structure has been proposed (Wei et al. [5]). The primary objective of the present brief is to examine the issue of the structure of turbulent wall-bounded flow, and to comment in particular on several recent papers from researchers at the University of Utah [5–7]. The question is important because the mean-velocity and higher-order statistics profiles and their proper scaling can be gleaned from knowledge of the flow structure. Reliable knowledge of the flow structure might be a powerful base for turbulence modeling in general, but also provides a fundament for engineering tools such as the friction-law for pipe flows.

Following the idea advanced by Klewicki and his colleagues in a sequence of papers [5–7], where the authors advocated a meso-scale normalization intermediate to the traditional inner and outer scales, we search in the present paper for multiplicity of overlap regions or structures in the canonical wall-bounded flow. To identify potential overlap regions, we analyze the continuity, mean-momentum and turbulence kinetic energy transport equations using channel DNS data.

We will restrict ourselves here to the so-called canonical turbulent wall-bounded flows. These can be fully-developed boundary layers, or pipe or channel flows that are steady in the mean, incompressible and two-dimensional in the mean. In general, the canonical wall-bounded flow is not influenced by freestream turbulence, wall roughness, wall heating/cooling, Coriolis/centrifugal forces, or any kind of streamwise wall curvature. The flow should be fully turbulent meaning for a turbulent boundary layer the minimum Reynolds number of $Re_{\Theta,\min} \approx 350–730$ should be exceeded. Turbulence achieved by natural transition or tripping devices should commence far upstream. For turbulent boundary layers, canonical also means they are not complicated by any streamwise pressure gradient.

II. Structure Tool

Beside the analysis of high-quality data and ad hoc theoretical considerations, it is possible to employ the governing equations to learn more about physical phenomena. Imagine any physical process that is described by a set of equations. It is always possible to write these equations as sums of terms:

$$\begin{aligned} T_{1,1}(x_j) + T_{1,2}(x_j) + T_{1,3}(x_j) + \ldots + T_{1,k}(x_j) = 0 \\ T_{2,1}(x_j) + T_{2,2}(x_j) + T_{2,3}(x_j) + \ldots + T_{2,k}(x_j) = 0 \\ T_{n,1}(x_j) + T_{n,2}(x_j) + T_{n,3}(x_j) + \ldots + T_{n,k}(x_j) = 0 \end{aligned} \tag{1}$$

Here x_j denote all independent variables or arguments of the equations. Selecting one term of interest in each equation, $T_{i,j}(x_j) = T_{i,S}(x_j)$,‡ and dividing each particular equation by its chosen term leads to

$$\sum_{j=1}^{k} \frac{T_{1,j}(x_j)}{T_{1,S}(x_j)} = -1; \qquad \sum_{j=1}^{k} \frac{T_{2,j}(x_j)}{T_{2,S}(x_j)} = -1; \qquad \ldots; \qquad \sum_{j=1}^{k} \frac{T_{n,j}(x_j)}{T_{n,S}(x_j)} = -1 \tag{2}$$

The ratios $T_{i,j}(x_j)/T_{i,S}(x_j)$ can then be interpreted as ratios of certain

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‡Alternatively one certain term from the whole system of equations can be selected as term of interest.

forces, fluxes or moments, whose sum is minus unity. Now consider the absolute value of a certain ratio. In case that this value is very close to zero, then the effects represented by the numerator is negligibly small compared to the term of interest $T_{i,S}(x_j)$. For an absolute value of the ratio much larger than unity, the force, flux, or moment represented by $T_{i,S}(x_j)$ is negligibly small. If the absolute value is close to unity the compared effects are similar. Certain regions within the domain where the system (1) is valid can then be qualified by these ratios.

III. Structure of Wall-Bounded Flows

To investigate the structure of the canonical wall-bounded flow we apply the tool described in the previous section. The following three mean equations are employed for the present analysis: 1) continuity equation, 2) streamwise mean-momentum equation, and 3) transport equation of turbulence kinetic energy.

Let us first consider the continuity equation for the mean flow of a two-dimensional boundary layer:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Selecting the second term of (3) as $T_S(x_j)$ and dividing by this term results in

$$\frac{\partial u/\partial x}{\partial v/\partial y} = -1 \tag{4}$$

Interpretation of (4) leads to the rather simple and surprising result that a wall-bounded flow consists only of one layer, which implies that the flow does not have any layered structure. This is true with respect to the mass flux in any incompressible turbulent boundary layer, channel flow, or pipe flow. Based only on the kinematics, no region of the flow could be singled out as to qualify a particular structure. The reason is simply that any structure of a canonical wall-bounded flow must be related to the viscosity, which is not represented in the continuity equation.

Applying the structure tool to the nondimensionalized mean-momentum equation of pipe and channel flow, we obtain

$$\frac{1}{\delta^+} + \frac{d^2u^+}{dy^{+2}} - \frac{d\langle uv\rangle^+}{dy^+} = 0 \tag{5}$$

where the "+" superscript indicates normalization with the inner velocity and length scales, u_τ and ν/u_τ, respectively. With the gradient of the Reynolds shear-stress as the term of interest $T_{1,S}(x_j)$, we obtain, as was first successfully demonstrated by Wei et al. [5], and extended by Wei et al. [6],

$$\frac{1/\delta^+}{d\langle uv\rangle^+/dy^+} + \underbrace{\frac{d^2u^+/dy^{+2}}{d\langle uv\rangle^+/dy^+}}_{\mathrm{VR}} = 1 \tag{6}$$

Figure 1c shows the ratio of the gradient of the viscous stress to the gradient of the Reynolds shear-stress, VR, for the DNS channel flow data of Kim et al. [8] and Abe et al. [9] The Kármán number for these two data bases ranges from $\delta^+ = 180$ to $\delta^+ = 640$. The mean-velocity and Reynolds-stress profiles are plotted in Figs. 1a and 1b, respectively. The detailed analysis of these and other data provided by Wei et al [5]. leads to a four-layer structure consisting of 1) inner viscous/advection balance layer (VR $\ll$ 1), 2) stress-gradient balance layer (VR $\cong -1$), 3) viscous/advection balance meso-layer (singularity), and 4) inertial/advection balance layer (VR $\cong$ 0).

In a third trial scheme, the structure tool is applied to the equation of turbulence kinetic energy. For fully-developed channel flow this equation reads (Pope [10])

$$P - \varepsilon + \nu\frac{d^2k}{dy^2} - \frac{d}{dy}\left\langle\frac{1}{2}\nu u\cdot u\right\rangle - \frac{1}{\rho}\frac{d}{dy}\langle\nu p'\rangle = 0 \tag{7}$$

Here, P denotes the turbulence kinetic energy production, ε is the pseudodissipation, the third term is the viscous diffusion of the turbulence kinetic energy, the fourth is the kinetic energy convection, and the last is the pressure transport. Selecting the pseudodissipation ε as the term of interest $T_{1,S}(x_j)$, dividing (7) by this term, and regrouping leads to

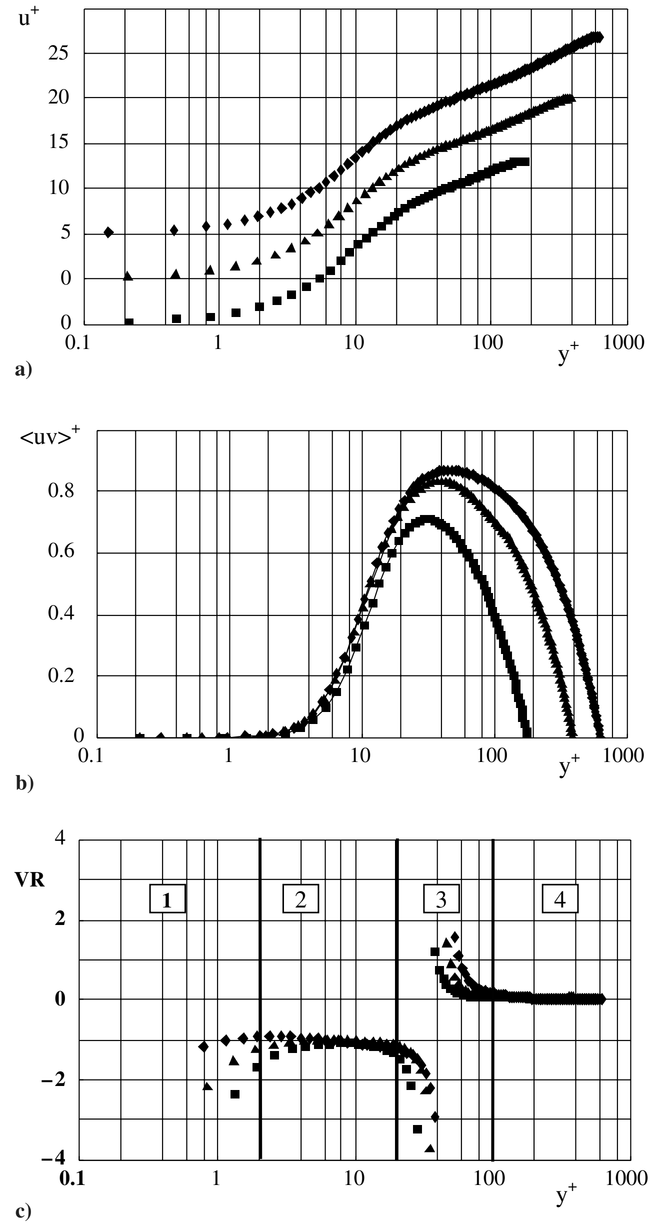

Fig. 1 Structure of turbulent channel flow. a) Mean-velocity profile second and third profile are shifted by a Δu^+ of 5 and 10, respectively; b) Reynolds shear stress; c) structure according to Klewicki et al. [7]. DNS data from Kim et al. [8], ■ $\delta^+ = 180$ and ▲ $\delta^+ = 395$; Abe et al. [9], ◆ $\delta^+ = 640$.

$$\frac{P}{\varepsilon} + \frac{\nu(d^2k/dy^2)}{\varepsilon} - \frac{(d/dy)\langle\frac{1}{2}\nu u\cdot u\rangle}{\varepsilon} - \frac{(1/\rho)(d/dy)\langle\nu p'\rangle}{\varepsilon} = 1 \tag{8}$$

The left column of Fig. 2 depicts the terms of Eq. (7) for the DNS data of Kim et al. [8] and Abe et al. [9], in the Kármán number range of 180–640. The four ratios obtained from Eq. (8) are plotted in right column of Fig. 2. Because of the fact that in the normalized transport equation of turbulence kinetic energy four ratio-terms occur instead of two as in the normalized mean-momentum Eq. (6), a comparison of these terms with unity is not possible. The visible structure is therefore much more complex than the one acquired based on the momentum balance (5).

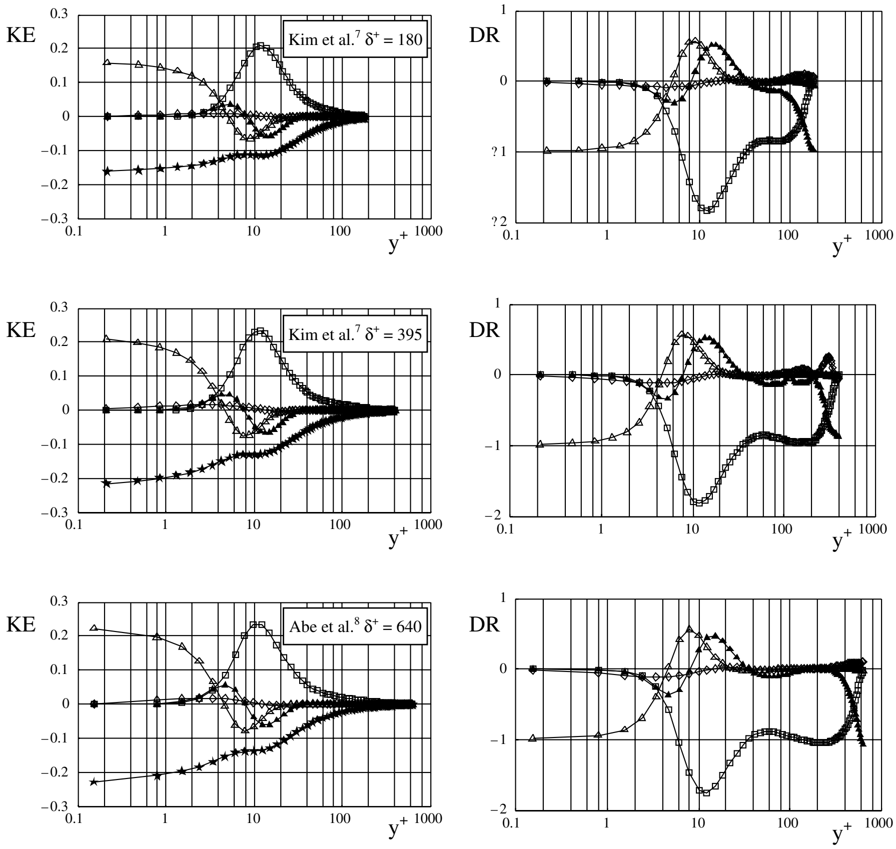

Fig. 2 Structure of turbulent channel flow. Left: terms of the turbulence kinetic energy equation (KE), Eq. (7); ★ pseudodissipation, □ production, △ viscous diffusion, ▲ kinetic energy convection, ◇ pressure transport. Right: dissipation ratios (DR); □ production/pseudodissipation, △ viscous diffusion/pseudodissipation, ▲ kinetic energy convection/pseudodissipation, ◇ pressure transport/pseudodissipation.

The only layer that can be identified immediately and that is in accord with the analysis from the mean-momentum equation is the inner viscous/advection balance layer (right column of Fig. 2). Here, almost no turbulence production occurs and viscous diffusion is virtually entirely compensated by viscous dissipation. Above this region, a multilayer structure occurs. Several layers individually characterized by a certain ensemble of ratios following from the equation of turbulence kinetic energy can be identified. Other equations may as well be analyzed and more layers may be identified. The number of layers clearly depends on the equation inspected, and no one equation holds superior. The physical arguments originally advocated by Prandtl [1], Kármán [11] and Millikan [2], who proposed a two-layer approach where the boundary layer is separated into two distinct albeit overlapping zones, are therefore superior to the more mathematical scheme advocated by the University of Utah research team [5–7].

IV. Conclusions

In this brief, we have used the continuity, mean-momentum, and turbulence kinetic energy transport equations to search for structure in the canonical turbulent wall-bounded flow. From the presented analysis the following conclusions are drawn:

1) The structure obtained for wall-bounded flows obviously depends on the equation analyzed. This seems to be plausible because each of the employed equations, conservation of mass, momentum, or balance of turbulence kinetic energy, has a different physical meaning. However, because these equations are not inconsistent with one another, the found structures should also be none contradictory, which is clearly not the case, casting doubt on the whole exercise.

2) None of the analyzed equations supports the two-layer approach directly. However, it is already known from early heuristic investigations and experimental work [1,2,11] that two regions exist in turbulent wall-bounded flows: close to the wall the no-slip condition enforces a viscosity-dominated region, and in the outer region, where Kármán's defect law is valid, inviscid turbulence stresses dominate.

To summarize, the two-layer structure model that is used for composite expansions [12] is an appropriate tool to describe the mean features of a turbulent wall-bounded flow. This includes canonical turbulent boundary layers, and channel and pipe flows. The two-layer structure model admits the universal logarithmic law as well as the more general, Reynolds-number-dependent log law derived by Buschmann and Gad-el-Hak [13]. The former law is valid, strictly speaking, for infinite Reynolds (or Kármán) number, whereas the latter is applicable over a broad range of Kármán numbers from moderate to infinite.

Analyzing the continuity, mean-momentum, and turbulence kinetic energy transport equations using channel DNS data, it was shown that the classical two-layer approach seems to be still a powerful tool to describe and model this type of flow. It is physically most convincing, and for practical purposes such as, for example, deriving the mean-velocity profile, the most efficient approach.

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